A Theoretical Physics Primer

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A Theoretical Physics Primer

Analytical Tools

by

Hermann Schulz

1st edition

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Preface

Those brave souls taking up physics for study and an introductory textbook have one thing in common: they face the same problem — although from different vantage points — the lack of a yet to be established convention on how to communicate. Everyday English is too imprecise, students come from school prepared to widely different degrees and there are grossly misguided ideas concerning what studying physics is all about. But we are all human beings, have all been for a walk in the fields, have marveled at the night sky, can visualize things with our eyes closed, possess the ability to wonder and are familiar with the question "Why ?". Doing Physics means to never cease asking this question.

Unraveling the processes of Nature takes place at a desk. Understanding means reduction to phenomena already known. This is done with equations, sketches, and calculations. Every symbol that appears on paper carries meaning. The art of understanding develops from the characters in formulae just like notes in a musical score can be turned into music. The art of understanding is a practical skill; its tools are the symbols in the formulae of "calculation with meaning". The object of this book is to explicate these symbols and to make their meaning transparent. It can accompany the reader only so far; only by trying things out oneself, by practicing, practicing and practicing more, will the decoder of notes become a pianist.

No particularly advanced degree of school knowledge is presumed. What an angle is, e.g. or why the theorem of Pythagoras holds will be explained. Perhaps — and that would be nice — parts from the first few chapters of the book may be used by teachers and students in advanced classes in high schools. In subsequent chapters other aspects will take precedence: efficiency (brief is beautiful; visualizing things saves time), elegance (hopefully; else, try to improve on it!) and the ability to discriminate between fundamentals, derivations and specialized applications (only in this way can one cope with the by now enormous field of physics). This places high demands now on the "reader without prior knowledge": the ability to reflect, imagination, being honest with oneself and a tremendous desire to write down by oneself, try out and improve all ideas accessible by the formulae — until one acquires the feeling of having invented them on one's own.

The book is based on a course for first–year students at Hannover University/Germany (lecture and exercises) under the heading *Calculational Methods of Physics.* This title is an overstatement. Formerly it was called *Supplementary*

Mathematics Course. Nothing in this title was quite correct. But everyone knew what it meant. With the title of this book it is just the opposite. It is correct — but one doesn't quite understand it. That is because it contains two words that are foreign. It is not uncommon for these two words

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to be thoroughly misunderstood. Let us first "translate" the noun. What is remarkable about physics is that it exists at all. And it has only existed – in the proper sense of the term – for about 300 years. We have known about regularities in Nature's processes for a long time — ever since man was able to record and communicate his observations. Under identical conditions processes recur in exactly the same manner. Nature behaves mathematically. What was really exciting was the realization that there is *unity* in the mathematics of these processes. There is only *one* mathematics involved, valid for all phenomena. This may sound incredible. One is entitled to have doubts (they will be removed during one's studies). If the statement is correct, though, then feelings of awe are called for at this point. That a unique "nature–mathematics" does exist is Nature's wonder No.1. Mathematics is based on axioms (a few initial statements that determine everything that follows from them). The axioms of the "nature–mathematics" are called *first principles*. If we know the world's first principles we can — in principle — understand all its phenomena. Understanding is now equivalent to reduction to these axioms. The initial first principle (it was incomplete and not quite right, but after all it was the first) was formulated by Newton in 1687.

We shall now try to give a definition of *physics* (in a way not to be found in any dictionary):

> Physics is the (one) fundamental natural science that, on one hand, looks for the (small number of correct and exhaustive) *first principles* of "nature–mathematics" and, on the other hand, seeks to understand the phenomena of Nature by demonstrating them to be inevitable consequences of such principles (as far as they are already known).

The back side of this definition is somewhat malicious. As soon as one ceases to have anything to do with Nature's first principles, one is no longer dealing with physics at all. The reader is invited to reflect on how well our definition differentiates physics from other natural sciences. It is not "arrogant" but certainly very high-brow. Biologists and chemists can rightly reply that we do not yet understand even a blade of grass or the properties of water. For the time being that is still too difficult.

Physics and calculating are thus inseparable. Mathematicians do mathematics; physicists do "nature–mathematics". At worst, the former may contain a logical error. The latter, in contrast, can also be wrong because it does not conform to the actual behavior of Nature. Physics thus has two supreme judges: logic *and* reality. Maybe that is why it is commonly regarded as being "difficult". One can be ridiculed all too easily for producing a solution to an exercise that is (almost) completely logical but nonetheless wrong. How come? There were two solutions to the problem, for instance, but only one made sense.

Now it is easy to understand the adjective. It refers to a division of labor. Experimental physicists spend their working lives closer to natural phenomena and theoretical physicists closer to logic — and exclusively at their desks. The choice of the word "theory" is unfortunate. It seems to imply that one is free to invent the way the world is constituted, or that physics is just one way to interpret the world among many. No, every detail of our current knowledge of "nature–mathematics" has been examined and confirmed by thousands of physicists. Proof of even the slightest deviation will lead to a Nobel Prize. Theoretical physics comprises the most solid statements that man is able to express about Nature.

There is only one way to get to the heart of things. The inner harmony of Nature is accessible only to those who have mastered the art of "calculation with meaning". This means to have a firm grasp of its

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to make use of them, to work and think in terms of them. These tools and their symbols are also those of mathematics. They appear on paper. The comparison with the pianist fails here because now *everything* takes place on paper. We are both composer and pianist. To each of the following 16 chapters one may naturally assign a typical symbol in a free and easy manner. In "cuneiform script", the contents, for example, looks like this:

$$\begin{gathered} \vec{\ } \qquad \dot{\vec{\ }} \qquad m\ddot{\vec{r}} \qquad DHD^T \qquad \mathrm{e}^x \qquad \int \qquad Ly = f \\ \nabla \qquad \oint \qquad \mathrm{e}^{tD\Delta} \qquad \vec{E} \times \vec{B} \qquad \mathrm{e}^{\mathrm{i}kx} \qquad \delta S \qquad -\sum p \ln(p) \\ mc^2 \qquad \mathrm{i}\hbar\dot{\psi} \qquad . \end{gathered}$$

These symbols (and many more) are like building blocks out of a construction kit. They are capable of causing an incredibly large amount of work. By means of these building blocks Nature can be partially "reconstructed" and — even more importantly — predicted.

Theoretical physics is something one *does*. I sit at my desk and consider a certain natural process which I should like to comprehend. So I start to *draw*. That is a good thing. We did not acquire the ability to draw directly by Darwinian selection so some effort is required in order to actually do it. Sketches nearly always have to be improved. The same applies to calculations. So I use a pencil. It is well suited to the way we work: noting something down — reflecting — revising. I want to be able to erase. I need to feel free when I draw and in order to draw, and when I calculate in order to facilitate the next step of the calculation.

Another advantage of the pencil is that it can be sharpened. If necessary one can easily distinguish between four different letter sizes (imagine a subscript with index on index on index). We write on *blank* paper. The reader can easily prove to himself (and his old school-teacher) how much squared paper contravenes the way we work. The world is not made up of squares and all types of templates are detrimental for us. At venerable universities, lecturers are often reluctant to give this type of advice. However, when you study physics it is particularly important to vary your patterns of thinking and proceeding until you have found the ones that suit you best.

All the analytical tools that are dealt with in the following 14 chapters will indeed be constantly required in the course of your studies. The majority (99%?) of the calculations used in the natural sciences are based on them. At the end of each chapter, there is time for contemplation and putting things into a general perspective (first work, then play). The character of a training program (lectures *and* exercises) has been preserved as far as possible. So one will find references (at unexpected places) in the text to the home exercises contained in part IV that now can — and must — be mastered. They demarcate the material for a week. If this material seems unduly large, it is because the book goes beyond the scope of the lecture course.

The exercises are small research projects. They are to be worked out individually and unassisted. The moment of truth will come in Part IV. Please: Don't ever complain that it took "15 hours" to solve a particular problem. It will only prompt a wry smile and comments such as: "Was the radio playing ?", "Oh yes, my last problem took 150 hours and a sleepless night" or "Then you just still needed to spend 15 hours on it". And, without quotation marks: no time spent on exercises is ever wasted. They *are* your course.

Good luck !

I am very grateful to Dr. A. Ziegler (APL in Osnabrück/Germany) and to A. A. Ludl (PhD in Paris). A. Ziegler translated Preface, Contents, section 1.1 and chapter 9, chapters 15 and 16 were translated by A. A. Ludl. Since the remaining 280 pages were compiled by myself, they could contain one or the other unusualness in English usage. If so, I apologize for that.

Thankfully, Marina Forlizzi and Neil Ashby detected and corrected various errors throughout the main text.

I also wish to thank Klaus Horn (Verlag Europa–Lehrmittel) for his unhesitant assistance during the production of the book.

Hannover, September 2015 *Hermann Schulz*

Contents

Part I

First Semester

1 Vectors

First steps are easy. Whenever we want to describe some process of nature (describe only : no physics yet), we will soon be forced to specify directions. Signposts come to mind — we need **arrows**. An everyday example demonstrates how much we need them.

We are standing at the corner of a soccer field. I am pointing in a certain direction towards the ball (more precisely : towards its center). My pal standing next to me sees the ball too, but he looks in a slightly different direction. The ball is moving in yet another direction. It is also spinning; its instantaneous spin axis has a direction as well. In addition the ball is attracted by the Earth; it feels a downward force, i. e. towards the center of the Earth. Wind is blowing from direction ...... Raindrops are falling in direction ...... At any moment the curved path of the ball's center lies in a plane that can be characterized by its (orthogonal) direction. A floodlight (which we see in direction ...) sends its rays in direction ... towards the ball. The ball looks shiny. Hence, there are light rays coming from a spot on its surface towards my eye. In such a ray, an electron of a gas atom experiences an alternating force perpendicular to the ray, i. e. in the "direction of polarization" of the light wave. And so on.

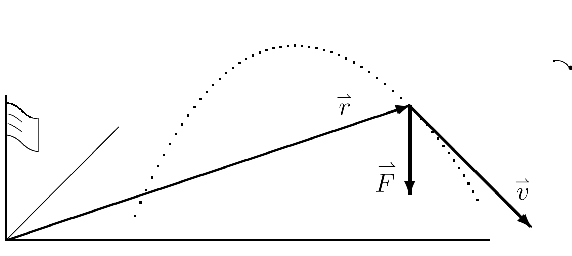

Figure 1–1 . Three everyday arrows: position, velocity, and force

Maybe you have noticed here a little human deficiency : My neighbor does not quite realize where I am pointing. "A crow", he says. Indeed there was a bird on the straight line through his nose and my fingernail. If only we were point–like creatures and could stretch our forefingers as much as we liked. Still we can do that in our imagination. And we can draw it, too, on a large drawing board.

1.1 Direction and Magnitude

We call an arrow that points from an agreed–upon point of reference, the **origin**, to any point of momentary interest within a physical process, a **position vector**. As abbreviation we write the letter r with an arrow on top of it. In hand writing one should use a half arrow $\rightharpoonup$, i. e. $\vec{r}$. This is common practice, efficient, and perfectly adequate. Creatures of habit capable of reasoning will adopt any minute improvement (whether it be in writing, thinking, speaking or calculating), and then adhere to it for life.

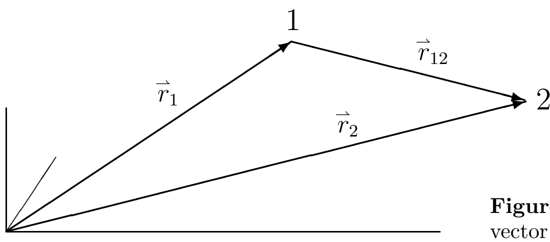

Figure 1–2 . A displacement vector and its two position vectors

An arrow that connects one point with another is called a **displacement vector** : $\vec{r}_{12}$ in Figure 1–2 . Thus, the position vector is a special form of a displacement vector, that always starts at the origin. If (as in Figure 1–1) we want to draw an arrow for velocity too, then we obviously have to specify the scale, i. e. how many meters per second are to correspond to 1 cm on paper (of the drawing board). The length of the arrow, translated if necessary into units of (e. g.) velocity, is called the **magnitude** of the vector, and is written

$$|\vec{r}\,| = r \quad , \qquad |\vec{v}\,| = v \quad , \qquad |\vec{F}\,| = F \quad .$$

Incidentally, it is not yet necessary at this point to think about units of force.

So an arrow has direction, magnitude, and a starting point. In turn, the latter has a position vector of its own. We shall say : "At $\vec{r}$, the ball has velocity $\vec{v}$" or "at $\vec{r}$, the ball is accelerated by force $\vec{F}$". We realize that whatever can be expressed by one arrow and its starting point, can as well be expressed by two arrows. Proceeding this way nothing needs to be said about the starting points of either arrow. From now on we shall exclusively deal with these, endearingly humble, arrows : only their magnitude and direction have to be given. The point of this trick will become clearer still if we imagine a flow of water (Figure 1–3) : We can (in any reasonable domain) specify the velocity of an alga (if one happens to be there) at *any* position. We write $\vec{v}(\vec{r}\,)$ and say "vee of arr", and "$\vec{r}$ is the space variable". Such an arrangement is called a field : the velocity field of the flow, or perhaps the force field of the Earth.

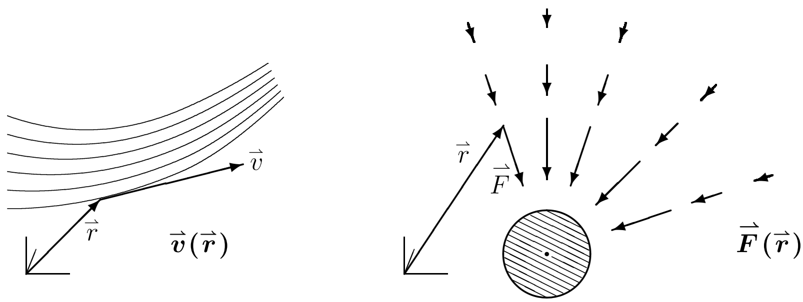

Figure 1–3 . Velocity field and force field

Maybe it is about time to clarify what a **vector** is supposed to be :

Preliminary definition

> Vectors are arrows with respect to magnitude and direction, for which it makes physical sense to multiply them with a number, and to add them up. (1.1)

This sentence sounds peculiar and too (?) pictorial. Probably it is not precise. We can't do it any better yet! The proper definition is given at the end of section 4.1. But let us emphasize one thing now : A physicist's notion of a vector is different from a mathematician's. We place particular emphasis on describing reality. An arrow can be constructed from wood or wire, and positioned to the right place by means of a scaffolding. It is still there even when nobody is looking. Ants crawl over it, and raindrops run along it. Because arrows are real (and vectors are supposed to be arrows), physicists demand somewhat more : The components of a vector should change in a certain way on transformation to a rotated frame of reference.

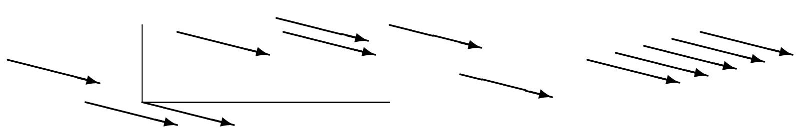

Figure 1–4 . Several different ways of depicting the same vector

For the time being we only understand the first line of (1.1). All arrows with identical magnitudes and directions form the same vector : Figure 1–4 . Or : A vector is the totality of an infinite number of arrows with the same magnitude and direction. Thus we can displace an arrow parallel to itself anytime. It continues to be a representative of the same vector ! In particular we can always select the arrow that starts at the origin, perhaps in order to measure the vector components.

Multiplication with a number

What is meant by this is shown in Figure 1–5 . Minus 2.7 times a vector is another vector, which points in the opposite direction and is 2.7 times as long as the original one.

Figure 1–5 . Multiplication of a vector with a number

If we multiply a displacement vector (1 meter long) with 1/(1 second), we get a velocity vector with magnitude 1 m/s. If we multiply any vector with 1/(its magnitude), we get a **unit vector**.

$$\frac{1}{a}\,\vec{a} = \vec{e} \quad , \quad |\vec{e}\,| = 1 \qquad \text{or:} \quad \vec{a} = a\,\vec{e} \quad . \tag{1.2}$$

The phrase "vector = (its) magnitude times (its) unit vector" is therefore always correct. People who are very much used to components may need an occasional reminder (when doing exercises, for instance) that it is always possible to think along these lines as well.

When we do physics and write a number without something following it (e. g. 1) this is usually wrong. "one of what ?? — one apple ? one meter ? one second ?" Most variables have a **dimension**, i. e. they are "a length" or "a time" etc. A unit vector however has indeed magnitude 1 (without anything after it), and 1.7 times $\vec{e}$ has magnitude 1.7. The agreement, common amongst mathematicians, to translate meters into numbers without a dimension, is found here too. Incidentally the author is incapable of drawing a unit circle. Can you do it ? We always end up with a circle having a radius in centimeters. But to specify a translation rule — this is possible.

Addition of two vectors

We demonstrate this using Figure 1–2 . $\vec{r}_1$ **plus** $\vec{r}_{12}$ is a vector as well, namely $\vec{r}_2$. Hence, every displacement vector can be rewritten in the form $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$ with the position vectors of the finishing and starting points. This definition can immediately be extended to any two vectors as long as they have the same dimension (i. e. they may be drawn on the same drawing board, with a specified translation rule) : Place $\vec{b}$ at the finishing point of $\vec{a}$, and then $\vec{a} + \vec{b}$ is the vector from the starting point of $\vec{a}$ to the finishing point of $\vec{b}$. Figure 1–6 demonstrates that the order of the two vectors does not matter: $\vec{a} + \vec{b} = \vec{b} + \vec{a}$. If the addition of a set of vectors leads us back to the starting point, then the result has magnitude zero: this is the **null vector**.

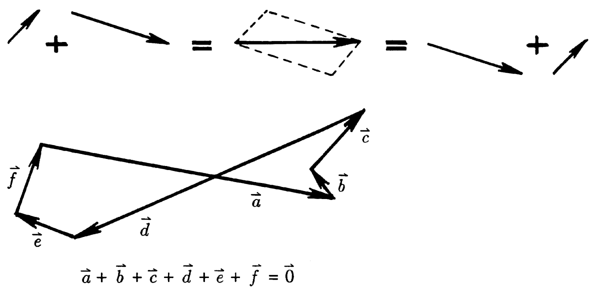

Figure 1–6. Addition of vectors, and how the null vector can result from it

We now understand (1.1) completely as far as displacement vectors are concerned. Only the term "physical sense" is still a bit grandiose. It is a different matter if we now ask whether velocities or forces are vectors in the sense of (1.1).

Are velocities vectors?

It makes sense to multiply them with a number. Figure 1–7 shows what is meant by adding a velocity $\vec{v}$ (an ant relative to a conveyor belt) to another velocity $\vec{u}$ (the belt). Does forming $\vec{u} + \vec{v}$, as described geometrically above, yield the correct total velocity $\vec{w}$? The answer is "Yes". Consider a specific time interval

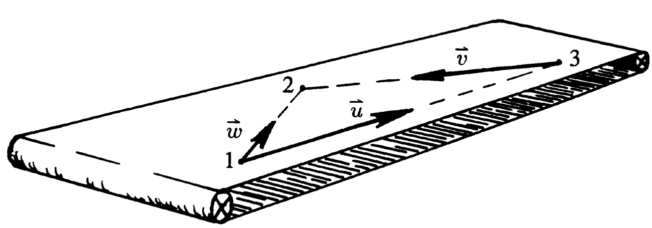

Figure 1–7. How velocities are added

Δt. Let $\vec{r}_1$ and $\vec{r}_2$ be the ant's positions at the start and the finish of this time interval, respectively. It is obvious that the ant can also reach $\vec{r}_2$ if it first sits still during Δt, thereby reaching $\vec{r}_3$, and then walks to $\vec{r}_2$ in Δt on a stationary belt: $\vec{r}_{12} = \vec{r}_{13} + \vec{r}_{32}$. We divide this equation by Δt and get $\vec{w} = \vec{u} + \vec{v}$. Thus it does indeed make physical sense to add velocities geometrically. Velocities *are* vectors.

Are forces vectors ?

We tie two strings to a spring balance (Figure 1–8) and pull one string with $\vec{F}$ and the other with $\vec{K}$. If we were to only pull one instead, with $\vec{F}+\vec{K}$, the spring balance would show the same reading and point in the same direction. We should not consider this fact a matter of course. It is a statement about Nature. Experimental result : Forces *are* vectors.

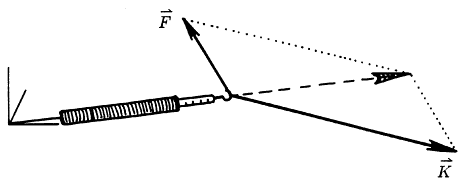

Figure 1–8 . How forces are added

The hook is at rest; not only are the forces $\vec{K}$ and $\vec{F}$ pulling on it but so too is the spring itself, namely with a force $-(\vec{K}+\vec{F})$. The sum of the forces is zero. The converse is also true, and quite generally : if a point is at rest, then it is not accelerating : Sum of forces = Mass times no acceleration = $\vec{0}$.

Are rotations vectors ?

Take the book lying in front of you and rotate it until it is lying crossways. You have rotated it around an axis perpendicular to the table, through a right angle. Point the thumb of your right hand upwards. The other (hand clenched) fingers will automatically indicate the sense of rotation (*right–handed* here). Thus a rotation (length = angle) may be characterized by an arrow. Now perform two rotations in series, first as in the left hand side of Figure 1–9 , then as in the right hand side. The two orientations the book ends with are different. This experiment should actually be carried out with the very book from whose chapter 1 it is taken, i. e. [*Berkeley* , 1]. It is listed in the Bibliography. Thus finite rotations are *not* vectors. Definition (1.1) — what it includes, and what it excludes — is now clear.

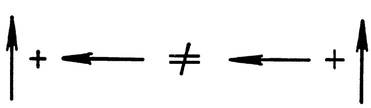

Figure 1–9 . Finite rotations are not vectors

Components

So far we could understand and formulate relationships without referring to coordinate axes (the edges of the soccer field, the flag pole). This fills us with